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Some remarks on the critical behaviour of superconducting percolation networks

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Abstract. Recent relations that relate the exponent s of superconducting percolating networks to other percolation exponents are discussed. We analyse the available exact and numerical results for s to gain insight into the possible relation between s and the geometrical exponents of percolation and the structure of superconducting percolation networks. The results are given geometrical interpretation. We also discuss the random walk statistics of the 'termite' which executes a random walk on the superconducting percolation networks. In particular we propose an expression for the mean number of distinct sites visited by the termite and interpret it in terms of the statistics of a random walk with a transition probability whose variance is infinite.

1. Introduction

Conductivity of random networks near the percolation threshold p_c has been much studied because transport processes and many other phenomena in disordered systems can be modelled by means of random networks of conductors. Near p_c the conductivity Σ of the network exhibits power-law behaviour. For a network with a fraction p of conducting bonds and a concentration $(1-p)$ of insulating bonds one has, $\Sigma \sim (p-p_c)^t$, whereas $\Sigma \sim (p_c-p)^{-s}$ for a network with a fraction p of superconductors (i.e. bonds with zero resistance) and a concentration $(1-p)$ of ordinary conductors. The critical exponents t and s are believed to be universal. An outstanding problem is whether there exist simple relations which relate t and s to the static exponents of percolation. t, which is a dynamical exponent, can be expressed as

$$t=(d-2)\nu+\zeta \tag{1}$$

where d is dimensionality and ν the exponent of percolation correlation length ξ_p. Here ζ is a universal exponent which describes the divergence of the resistance L of links in the 'nodes and links' model of the backbone (Skal and Shklovskii 1974, de Gennes 1976). In this model the backbone, i.e. the current-carrying part of the infinite cluster, is assumed to be made of nodes which are connected by long chains of several bonds called links. In the 'nodes and links and blobs' model of the backbone (see Stanley and Coniglio 1983 for a review) ζ represents the divergence of the resistance of one-dimensional chains along the backbone of the largest percolation cluster at p_c (Coniglio 1981). This means that in a transformation from the largest percolation cluster at p_c to its backbone ζ remains invariant.

In this paper we discuss the relation between s and the geometrical exponents of percolation. The exponent s plays an important role in the transport properties of disordered systems. It appears in the critical behaviour of the dielectric constant (Efros and Shklovskii 1976, Girannan *et al* 1981, Wilkinson *et al* 1983), the absorption coefficient of random metal-insulator composites (Bowman and Stroud 1984), in the conduction of binary metallic mixtures (Straley 1977, Fogelholm and Grimvall 1983) and in the viscosity of a gel (de Gennes 1979). By analysing the available exact results and the numerical estimates of s we will attempt to gain insight about the structure of superconducting percolation networks below p_c and the possible relationship between s and the geometrical exponents of percolation such as ν. Since this problem can be formulated as a random walk problem (de Gennes 1980) our analysis may enable us to gain some understanding about the properties of this random walk.

The plan of this paper is as follows. In § 2 we analyse the available results for the exponent s by using a modification of equation (1). Our analysis indicates that s might follow certain laws at various dimensionalities. The conditions under which these laws might hold and the geometrical interpretation of them are also discussed. In § 3 the analysis of § 2 is employed to discuss the properties of the random walk model of superconducting percolation networks. Section 4 relates the present problem to the concept of 'antifractal' recently discussed by Pandey (1984). The paper is summarised in § 5.

2. The analysis of the data

An equation similar to (1) can be derived for s. Just below p_c a percolating network is made of large superconducting clusters whose linear dimensions are comparable to ξ_p. These clusters offer zero resistance to an applied field E and therefore they play the role of nodes in the 'nodes and links' model of backbone above p_c. The resistance of the network is simply the sum of resistances of the connecting channels between these clusters. In the presence of E, the potential difference between two adjacent superconducting clusters is of order $E\xi_p$, although at low enough dimensions, because percolation clusters partially overlap (see below), this might not be precisely correct. To calculate the conductivity of the network one needs to know the volume fraction x that the connecting channels represent (Straley 1980, Coniglio and Stanley (CS) 1984). If the thickness of these connecting channels is finite, then x should be proportional to the ratio of surface to volume of superconducting clusters. However, one has to consider that part of the surface of a superconducting cluster which is not screened and is 'reachable' (Coniglio and Stanley 1984); the rest of the surface sites do not contribute to Σ. The precise scaling behaviour of x near p_c is therefore not known (although Coniglio and Stanley (1984) have made some attempt to study it) and hence we assume that $x \sim (p_c - p)^{\zeta'}$. Since the conductivity of the network is proportional to $x(E\xi_p)^2$ (i.e. the dissipated power), we obtain

$$s = 2\nu - \zeta' \tag{2}$$

where $\zeta' \neq \zeta$, for at least some range of d. Stephen (1978) also derived an equation similar to (2). However, he chose $\zeta' = \beta$, where β is the critical exponent of the strength of the infinite cluster. But his choice was shown by Kertész (1983) to misrepresent the physics of the problem. Moreover, Stephen's (1978) equation fails badly at all dimensions below six.

Let us investigate the behaviour of ζ' by using the most recent and accurate estimates of s. At $d = 1$ one has the exact result $s = 1$, which means that $\zeta' = 1$ as it should be. In two dimensions one must have $s = t$, as a result of duality (Dykhne 1970, Mendelson 1975, Straley 1977). The most recent estimates of s and t at $d = 2$ are as follows. Zabolitzky (1984), Hong *et al* (1984) and Lobb and Frank (1984) all estimated t to be about 1.3. Adler (1985) reanalysed the series expansion data of Fisch and Harris (1978) for t to take into account the effect of non-analytic confluent corrections and obtained $t \simeq 1.34$. Calef *et al* (1984) developed two-point Padé summation techniques to investigate the behaviour of the configurationally averaged Green function for a random walk on a site-disordered percolating network and obtained $t \simeq 1.38$, whereas Herrmann *et al* (1984) estimated that $s \simeq 1.3$. A Flory approximation (Family and Coniglio 1985) yields $t = \nu$ at $d = 2$. Thus one obtains $\zeta'(d = 2) = 1.29 - 1.37$ (where $\nu(d = 2) = \frac{4}{3}$) with an average of about 1.33. In three dimensions Herrmann *et al* (1984) estimated that $s \simeq 0.75$, while earlier Bernasconi (1978) gave $s \simeq 0.77$. Bowman and Stroud (1984) measured s experimentally and obtained $s \simeq 0.73$. Most of the other numerical estimates of $s(d = 3)$ reported in the literature lie inside this interval if their reported error estimates are taken into account. This means that $\zeta'(d = 3) = 0.99 - 1.03$, with an average of about 1.01, if $\nu(d = 3) \simeq 0.88$. For six- and higher-dimensional systems one has the exact result, $s(d \geqslant 6) = 0$, so that $\zeta'(d \geqslant 6) = 1$, as expected. We are not aware of any accurate estimate of s at any other dimensions. Thus at low dimensions the numerical values of ζ' are very close to those of ν, whereas in higher-dimensional systems ζ' appears to change very little and its value is about unity. (We remind the reader that ζ' should be monotonic above some certain dimensionality. Since ζ' is non-monotonic between $d = 1$ and some $2 < d_l < 3$, we expect it to be monotonic above d_l.) We may then observe that

$$s \simeq \nu \qquad d \leqslant d_l, \tag{3}$$

$$s \simeq 2\nu - 1 \qquad d \geqslant d_l. \tag{4}$$

Here d_l is the dimensionality which separates the two regimes, i.e. the dimensionality at which $\nu \simeq 2\nu - 1$. This yields $\nu(d = d_l) \simeq 1$, i.e. $d_l \simeq 2.65$. The existence of a critical dimensionality which separates two different regimes in each of which t and s may be related to other percolation exponents by different relations, has recently been argued by several authors (Aharony and Stauffer 1984, Coniglio and Stanley 1984, Sahimi 1984). Aharony and Stauffer (1984) and Sahimi (1984) argued that for the exponent t the dimensionality d is such that the fractal dimensionality d_f of the largest percolation cluster at p_c is exactly equal to 2. However, there is no reason to believe that d_l (if it indeed exists) should be the same for both exponents. Coniglio and Stanley (1984) also argued that for s the dimensionality d_l is such that $d_f = 2$. However, as we discuss below, their proposed relations for s are not likely to be exact and thus for s the dimensionality d_l may be different from what they suggested. Note that with (3) and (4), $d_f(d = d_l) \simeq 2.3$. Note also that equations (3) and (4) do not imply any discontinuity for s or ζ'. They simply mean that ζ' reaches its high-dimensional value just above d_l.

We do not make any claim about the exactness of (3) and (4). In the absence of ε ($\varepsilon = 6 - d$) expansion for s and with the present numerical estimates, it is impossible to check the validity of (3) and (4). However, because these equations are inaccurate they have several interesting consequences which we discuss now. If equation (3) holds exactly (i.e., if $s = \nu$), it would then agree with ε (here $\varepsilon = d - 1$) expansion of s given by Kirkpatrick (1977). It would also mean that the relation $s = \nu - \frac{1}{2}\beta$ which was proposed by Kertész (1983) (Coniglio and Stanley (1984) argued that it should hold for $d_f \geq 2$) cannot be exact, since at $d = d_l$ one must have $\nu = \nu - \frac{1}{2}\beta$, which is not possible. Straley (1980) has conjectured that $\zeta = \zeta'$ *at all* dimensions which means that $t + s = d\nu$. In three dimensions where $t \simeq 2$ and $s \simeq 0.75$, Straley's relation may not be satisfied. Moreover, the physical significance of ζ and ζ' seem to be different. However, if the relation proposed by Aharony and Stauffer (1984) and Sahimi (1984) for t for $d_f \leq 2$, $t = (d-1)\nu$, is exact, then Straley's relation is satisfied for $d_f \leq 2$ but not for $d_f \geq 2$. At very high dimensions Straley's relation is more accurate. For example, at $d = 5$ and with $t \simeq 2.7$ (Adler 1985) it yields $s \simeq 0.15$, whereas (4) gives $s \simeq 0.14$, if we use $\nu(d = 5) \simeq 0.57$. Ohtsuki and Keyes (1984) have discussed the significance of a fractal dimensionality d_{CA} which describes the effective area of the contact region between large superconducting clusters slightly below p_c (i.e., a fractal dimension which describes the connecting channels). They have shown that $d_{CA} \equiv d - 2 + s/\nu$ and have proposed that $d_f \geq d_{CA} \geq d_f - 1$, and $d_{CA} \leq d - 1$. Equations (3) and (4) suggest that $d_{CA} \simeq d - 1$ for $d \leq d_l$ and the inequalities hold strictly for $d \geq d_l$.

One should note that if (3) and the equation, $t = (d-1)\nu$ were to hold exactly at low dimensionalities, one would obtain $d_{CA} = d - 1$ for this range. This is the maximum value that d_{CA} can attain, i.e. its value in a Euclidean space. This is because d_{CA} for the present problem is equivalent to d_u, the fractal dimension that was introduced by cs to describe the unscreened surface sites M_u of fractal structures. M_u increases with the molecular diameter ξ as $M_u \sim \xi^{d_u}$, and thus for Euclidean objects such as hyperspheres, $d_u = d - 1$. The computer simulation results of Herrmann (1979) for two-dimensional percolation clusters below p_c show that the density profile, i.e. the probability that a given site belongs to a finite percolation cluster if it is at a distance r from the centre of mass of the cluster, is almost Gaussian. This means that these clusters have a very dense structure. If this density profile were exactly Gaussian, one would have $d_{CA} = d - 1$ (i.e., $s = \nu$), consistent with our observation. Moreover, at low dimensions the finite percolation clusters below p_c overlap stronger and stronger as p_c is approached, which means that two large superconducting clusters occupy partially the same volume if their centres of mass are close together. The effect of this interpenetration is to increase the contact area between two superconducting clusters and, therefore, we may expect that $\zeta' \geq \nu$. For $d \geq d_l$ where $\zeta' \simeq 1$, this inequality holds strictly. These observations do indicate that at low enough dimensions d_{CA} attains at least a value very close to its value in Euclidean space, i.e. $d_{CA} \simeq d - 1$, which is indicative of the rather dense structure of superconducting clusters.

3. The relationship with the 'termite' problem

We may reinterpret the data in terms of the random walk of the 'termite' (de Gennes 1980, Coniglio and Stanley 1984). To describe the physics of superconducting networks, de Gennes (1980) suggested that we consider a random walker, called a 'termite', which performs a normal random walk, i.e. a random walk in which the duration of each step is finite, when *off* the superconducting cluster but which moves instantaneously when *on* the superconducting cluster. This is of course because there is zero potential gradient (zero resistance) along a superconducting bond. De Gennes' model is sometimes called 'the stopwatch termite'. Since the only effect of superconducting cluster is to stop the clock, the trace of this termite is the same as that of a simple random walk. Bunde *et al* (1985) found, by numerical simulations and analytic arguments, that the diffusion coefficient D of this termite is given by $D = (1-p)^{-1}$.

Thus a severe drawback of this model is that there is no singular behaviour at p_c (except at one dimension).

More recently, more realistic random walk models of diffusion in superconducting percolation networks have been proposed. In particular, Bunde *et al* (1985) and Adler *et al* (AAS) (1985) have studied a random walk model whose diffusion coefficient has, in contrast with de Gennes' original 'stopwatch' termite, singular behaviour at p_c. Moreover, if R is the root-mean-squared displacement of the termite, one has anomalous diffusion for $R \ll \xi_p$. Thus one may introduce a fractal dimension d'_w for the termite, at time θ, through $R \sim \theta^{1/d'_w}$, similar to d_w of the ant which describes the conductivity of percolation networks of conductors and insulators. To obtain an expression for d'_w one has to realise that the termite's motion can start on any site of the network. It can also move to any site of the network. Thus the termite 'feels' the presence of superconducting clusters of all sizes. Hence d'_w must be an average quantity, the average being taken over the distribution of cluster sizes. For the ant problem this was investigated by Gefen *et al* (1983), who showed that if R is *averaged over all clusters* one obtains

$$d_w = 2(2\nu + t - \beta)/(2\nu - \beta). \tag{5}$$

For the termite problem the situation is somewhat controversial at present. In the model of CS (see also Bunde *et al* 1985) two types of jump frequencies, $\tau_o^{-1} = 1$ for the ordinary conductors and $\tau_s^{-1} \gg \tau_o^{-1}$ for the superconducting ones, are associated with the motion of the termite. At any given site the termite can choose any one of z nearest-neighbour bonds (z is the coordination number) for its next step. The transition probability for choosing bond i is given by $\tau_i^{-1}/(\Sigma_i \tau_i^{-1})$, where $\tau_i^{-1} = \tau_o^{-1}$ or τ_s^{-1}, depending on whether i is an ordinary or superconducting bond. The limit $\tau_s^{-1} \to \infty$ describes the termite of CS. The fractal dimension d'_w is predicted to be

$$d'_w = (2\nu - s)/\nu. \tag{6}$$

On the other hand, AAS proposed a different type of termite which can exit from a superconducting cluster from *any* site with *equal* probability. A similar model has also been discussed by Bunde *et al* (1985). To obtain d'_w for this model (the so-called 'Tel-Aviv' termite which is similar to 'Boston termite 2', see Bunde *et al* 1985) one may observe that the exponent $(-s)$ plays the same role for superconducting percolation networks as does the exponent t for the percolation networks of conducting and insulating bonds above p_c. Thus one might simply replace t in (5) with $(-s)$. Indeed, it is straightforward to repeat the analysis of Gefen *et al* (1983) for the termite problem. One finds that the only difference is the replacement of t with $(-s)$, which is of course because the diffusion coefficient D for the termite problem scales with $(-s)$ and not t. The final expression for d'_w is almost identical to d_w:

$$d'_w = 2(2\nu - s - \beta)/(2\nu - \beta). \tag{7}$$

Comparing (6) with (7) one may say that the absence of β in the CS formula means that the effect of different cluster sizes has not been taken into account. An elegant and somewhat different derivation of (7) has been given by Stauffer (1985) (whose work inspired the present argument) who starts from a general scaling equation which, in the appropriate limits, reduces to the ant or termite problems.

Unlike d_w equation (7) predicts d'_w to be a non-monotonic function of d. An examination of (7) using (4) shows that near $d = 6$ one has $d'_w \simeq 1$. We propose that $d'_w = 1$ for $d \geqslant 6$. We note that if the conjecture of Kertész (1983) (see also Sahimi

1983), $s = \frac{1}{2}(2\nu - \beta)$, were exact, equation (7) would predict $d'_w = 1$ at all dimensions, which we think is not plausible.

One can obtain expressions for $P_0(\theta)$, the probability of return to the origin at time θ, and $S(\theta)$, the mean number of distinct sites visited by the termite. Again we have to take into account the fact that the termite can start its motion on any site and thus $P_0(\theta)$ and $S(\theta)$ represent average quantities, the average being taken over the distribution of cluster sizes. For the ant problem these average quantities have been investigated by Angles d'Auriac and Rammal (1983) who obtained the following results for the ant

$$P_0(\theta) \sim \theta^{-d/d_w}, \tag{8}$$

$$S(\theta) \sim \theta^{-\alpha/d_w}, \tag{9}$$

where $d_w = 2 + (t - \beta)/\nu$ is the fractal dimension of random walks on the largest cluster at p_c and $\alpha = d - 2\beta/\nu$. For the termite problem we may repeat their analysis to obtain the corresponding expressions for $P_0(\theta)$ and $S(\theta)$. For example for the case of $p = p_c$ we may write

$$S(\theta) = \sum_n \rho(n) S_n(n, \theta) \tag{10}$$

where $\rho(n)$ is the probability that the motion of the termite starts on a cluster of n sites and $S_n(n, \theta)$ is the number of sites visited on such a cluster. $\rho(n)$ is related to the distribution of clusters of n sites (Stauffer 1979) at $p = p_c$, $n_s \sim n^{-(d/d_f+1)}$, by $\rho(n) = nn_s$. Here $d_f = d - \beta/\nu$ is the fractal dimension of the largest percolation cluster at p_c. For a given θ we find

$$S(\theta) \sim \theta^{-\alpha/d'_w}. \tag{11}$$

Similarly, for $P_0(\theta)$ we find

$$P_0(\theta) \sim \theta^{-d/d'_w}. \tag{12}$$

These are completely analogous to (8) and (9). Equation (12) has also been derived by AAS using a simpler argument which can be summarised as follows. The volume filled by sites visited after time θ is of order $R^d \sim \theta^{d/d'_w}$. Since all sites are equally probable the probability of return to the origin is predicted to be $1/R^d$, which results in (12). Thus the only difference between the ant and the termite, so far as the scaling laws for various quantities are concerned, appears to be the replacement of t with $(-s)$. Interestingly, the quantity α in (9) and (11) is what was originally thought to be the fractal dimension of the largest percolation cluster at p_c (Stanley 1977).

In one dimension (11) predicts that $S(\theta) \sim \theta$; for an ordinary random walk on a linear chain one has $S(\theta) \sim \theta^{1/2}$ (Dvoretzky and Erdos 1951). To interpret the termite result we note that Gillis and Weiss (1970) considered a random walk in which jump probabilities $p(r)$ for a vector displacement r were given by $p(r) \sim r^{-(1+\delta)}$, where $1 < \delta \leq 2$, so that the variance of $p(r)$ is infinite, i.e. the walker can take very long jumps. It was found that in one dimension

$$S(\theta) \sim \theta, \qquad 1 < \delta \leq 2. \tag{13}$$

In the termite problem of AAS, the random walker is placed at random on any site of the network, and then choose to walk to one of this site's nearest neighbours. If the new site is occupied by an ordinary conductor then one unit of time is recorded. If the new site is superconducting, a selection is made at random from amongst *all* the sites on the superconducting cluster to which this site belongs and the termite jumps to this site (no unit of time is recorded this time). As p_c is approached from below, larger and larger superconducting clusters appear and it becomes more and more likely that the termite takes very long jumps (and in one dimension at p_c it can travel the entire system in a single jump). Thus in this sense the termite problem of AAS and the random walker of Gillis and Weiss are similar. Another contributing factor to such a behaviour for $S(\theta)$ is that while the termite is visiting new sites on the superconducting cluster, the clock is stopped; thus we should expect $S(\theta)$ to grow with *the measured time* faster than $\theta^{1/2}$. We also believe that the growth of $S(\theta)$ with θ for the termite of AAS is faster than the corresponding case for the ant problem, even if (11) turns out not to be exact. For the same reason, one must have $d'_w < 2$. Note that for $d \geqslant 6$, we should have $S(\theta) \sim \theta^2$.

4. The relation with 'antifractals'

Random walks on most fractal systems such as percolation clusters have a fractal dimension $d_w > 2$. However, for some fractals and more generally, some disordered systems, it might be possible that $d_w < 2$ and hence one has a superdiffusive regime. Such systems were called 'antifractals' by Pandey (1984). An example is the motion of a particle in a static random potential for which $d_w = 1$ (Heinrichs and Kumar 1984a). Marianer and Deutsch (1984) have claimed that the result of Heinrichs and Kumar (1984a) is wrong and that Pandey's antifractal concept has no mathematical foundation (see also the reply of Heinrichs and Kumar 1984b).

The random walk of the termite on superconducting percolation networks below p_c is an example of a random walk on an antifractal. Here one has a well defined fractal structure and a random walk with $d'_w < 2$ for all $d < 6$. Thus the objection of Marianer and Deutsch (1984) to the notion of antifractal appears to us not to be well founded, at least in the case of superconducting percolation networks below p_c.

5. Summary

We have analysed the available exact and numerical results for the exponent s of superconducting percolation networks to gain insight about the possible relation between s and the geometrical exponents of percolation and the structure of superconducting percolation networks below p_c. We proposed an expression for the mean number of distinct sites visited and mean number of visits to the origin by the 'termite' which executes a random walk on superconducting percolation networks. We interpreted the result in terms of a random walk whose variance of its transition probability is infinite. We also pointed out that superconducting percolation clusters are well defined examples of 'antifractals' recently discussed by Pandey (1984).

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Note added in proof. A Bunde and D Stauffer (1985 Preprint) have argued that termites behave like ants for not too long times. They have also argued that the random walks of termites cannot be described in the general case by a two-argument scaling function. This claim is supported to some extent by the Monte Carlo results of Sahimi and Saddiqui (1985), but the matter is not yet settled.

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